Barcan Up the Wrong Tree

Andrew Parisi
andrew.parisi@uconn.edu

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Society for Exact Philosophy

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Necessitism, Contingentism, and The Barcan Formulas

NECESSITISM AND CONTINGENTISM

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Contingentism is the view denies necessitism.

The heart of necessitism is that the domains of all possible worlds are identical.

BARCAN FORMULAS

Williamson and others have taken the validity of the formula

$$\Diamond\exists x\varphi \leftrightarrow \exists x\Diamond\varphi \qquad \text{(BF+CBF)}$$

to be the point of disagreement between necessitists and contingentists.

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Thesis

BF+CBF are neither sufficient nor necessary for settling the debate between necessitists and contingentists.

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Thesis

BF+CBF are neither sufficient nor necessary for settling the debate between necessitists and contingentists.

Upshot: What is sufficient for settling the debate are the structural rules of a language that govern the behavior of names in modal contexts.

BQML

QUANTIFIED MODAL LOGIC

Language: Atoms $\cup\{\wedge, \neg, \Diamond, \exists\}$.

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Need to keep track of denoting and non-denoting terms

Hypersequents are sets of sequents flanked by sets of names

$$A \wr (\Gamma_1 \Rightarrow \Sigma_1); \ldots; (\Gamma_n \Rightarrow \Sigma_n) \int B$$

HYPERSEQUENTS

$$\varphi \quad \psi \quad \gamma \quad \theta$$

Sentences

HYPERSEQUENTS

φ	ψ	γ	θ
T	F	F	T

HYPERSEQUENTS

$$\begin{array}{ccc} \varphi, \theta & \Rightarrow & \psi, \gamma \\ T & & F \end{array}$$

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$$\begin{array}{ccc} \Gamma & \Rightarrow & \Sigma \\ T & & F \end{array}$$

HYPERSEQUENTS

$$(\Gamma \underset{w}{\Rightarrow} \Sigma)$$

HYPERSEQUENTS

$$\underset{w_1}{(\Gamma \Rightarrow \Sigma)} \quad ; \quad \underset{w_2}{(\Delta \Rightarrow \Lambda)}$$

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$$\underset{w_1}{(\Gamma \Rightarrow \Sigma)} \quad ; \quad \underset{w_2}{(\Delta \Rightarrow \Lambda)} \quad ; \quad \ldots ; \quad \underset{w_n}{(\Pi \Rightarrow \Theta)}$$

HYPERSEQUENTS

$$\begin{array}{cccccccc} A\wr & (\Gamma \Rightarrow \Sigma) & ; & (\Delta \Rightarrow \Lambda) & ; & \ldots ; & (\Pi \Rightarrow \Theta) & \int B \\ \text{Might denote} & w_1 & & w_2 & & & w_n & \text{Can't denote} \end{array}$$

HYPERSEQUENTS

$$\frac{\vdots}{A \wr (\Gamma \Rightarrow \Sigma); (\Delta \Rightarrow \Lambda); (\Pi \Rightarrow \Theta) \int B}$$

SAYS: There are no worlds where ...

CALCULUS: *BQML*

Structural Rules:

$$\text{Id}\ \frac{}{A \wr G; (\Gamma, \varphi \Rightarrow \varphi, \Sigma); H \int B}$$

$$\text{Id}_t\ \frac{}{A, t \wr G \int t, B}$$

CALCULUS: *BQML*

Structural Rules:

$$\text{Id}\,\frac{}{A \rmoustache G; (\Gamma, \varphi \Rightarrow \varphi, \Sigma); H \lmoustache B} \qquad \text{Id}_t\,\frac{}{A, t \rmoustache G \lmoustache t, B}$$

Weakening

$$\text{W}_S\,\frac{A \rmoustache G; (\Gamma \Rightarrow \Sigma); H \lmoustache B}{A \rmoustache G; (\Gamma, \Delta \Rightarrow \Lambda, \Sigma); H \lmoustache B} \qquad \text{W}_t\,\frac{A \rmoustache G \lmoustache B}{A, C \rmoustache G \lmoustache D, B}$$

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Negation:

$$\mathsf{L}\neg\ \frac{A \wr G; (\Gamma \Rightarrow \varphi, \Sigma); H \int B}{A \wr G; (\Gamma, \neg\varphi \Rightarrow \Sigma); H \int B} \qquad \mathsf{R}\neg\ \frac{A \wr G; (\Gamma, \varphi \Rightarrow \Sigma); H \int B}{A \wr G; (\Gamma \Rightarrow \neg\varphi, \Sigma); H \int B}$$

CALCULUS: *BQML*

- Modal Rules:

$$\text{L}\Diamond\ \frac{A \wr G; (\varphi \Rightarrow \ \ ); (\Gamma \Rightarrow \Sigma) H \int B}{A \wr G; (\Gamma, \Diamond\varphi \Rightarrow \Sigma); H \int B} \qquad \text{R}\Diamond\ \frac{A \wr G; (\Delta \Rightarrow \varphi, \Lambda); (\Gamma \Rightarrow \Sigma); H \int B}{A \wr G; (\Delta \Rightarrow \Lambda); (\Gamma \Rightarrow \Diamond\varphi, \Sigma); H \int B}$$

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- Quantifier Rules:

$$\mathsf{L}\exists\ \frac{A, t \wr G; (\Gamma, \varphi[t/x] \Rightarrow \Sigma); H \int B}{A \wr G; (\Gamma, \exists x\varphi \Rightarrow \Sigma); H \int B}$$

$$\mathsf{R}\exists\ \frac{A \wr G; (\Gamma \Rightarrow \varphi[t/x], \Sigma); H \int B \qquad A \wr G; (\Gamma \Rightarrow \Sigma); H \int B, t}{A \wr G; (\Gamma \Rightarrow \exists x\varphi, \Sigma); H \int B}$$

where t does not occur in the conclusion of $\mathsf{L}\exists$.

CALCULUS: *BQML*

- Modal Rules:

$$\text{L}\Diamond\ \frac{A \wr G; (\varphi \Rightarrow \quad); (\Gamma \Rightarrow \Sigma) H \int B}{A \wr G; (\Gamma, \Diamond\varphi \Rightarrow \Sigma); H \int B} \qquad \text{R}\Diamond\ \frac{A \wr G; (\Delta \Rightarrow \varphi, \Lambda); (\Gamma \Rightarrow \Sigma); H \int B}{A \wr G; (\Delta \Rightarrow \Lambda); (\Gamma \Rightarrow \Diamond\varphi, \Sigma); H \int B}$$

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where t does not occur in the conclusion of L$\exists$.

Fact

BF+CBF is valid according to *BQML*

Model Theory

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Models come with a set of worlds W, for each world w a domain d_w of individuals.

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Clause for Ʒ:

- $M, w, \sigma \models Ʒx\varphi$ iff there is a world v and $\sigma' \sim_x \sigma$ such that $\sigma'(x) \in d_v$ and such that $M, w, \sigma' \models \varphi$.

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Models come with a set of worlds W, for each world w a domain d_w of individuals.

Clauses for connectives and modal operators are standard.

Clause for Σ:

- $M, w, \sigma \models \Sigma x \varphi$ iff there is a world v and $\sigma' \sim_x \sigma$ such that $\sigma'(x) \in d_v$ and such that $M, w, \sigma' \models \varphi$.

Call this set of models $\mathcal{M}_C$.

SOUNDNESS AND COMPLETENESS

Let $\mathcal{M}_N \subset \mathcal{M}_C$ given by for all $w, v \in W$, $d_w = d_v$.

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Fact

BQML is sound and complete with respect to both $\mathcal{M}_C$ and $\mathcal{M}_N$

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Let $\mathcal{M}_N \subset \mathcal{M}_C$ given by for all $w, v \in W$, $d_w = d_v$.

$\mathcal{M}_C$ and $\mathcal{M}_N$ correspond to contingentist and necessitist theories of modal space.

Fact

BQML is sound and complete with respect to both $\mathcal{M}_C$ and $\mathcal{M}_N$

Upshot: BF+CBF is insufficient to distinguish between contingentism and necessitism

An Objection

OBJECTION

ARGUMENT

1. ⅀ is a quantifier.

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1. $\exists$ is a quantifier.
2. All quantifiers are ontologically significant.

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C *BQML* is not a contingentist logic (what exists according to *BQML* is all that's possible and that does not vary)

OBJECTION

ARGUMENT

1. Σ is a quantifier.
2. All quantifiers are ontologically significant.

C *BQML* is not a contingentist logic (what exists according to *BQML* is all that's possible and that does not vary)

RESPONSE: Premise 2 is false

Another calculus:

Add $\exists$ to the language.

CQML

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- Add $\exists$ to the language.
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- Add $\exists$ to the language.
- Change the structure of hypersequents:
  - A sequent is now a quadruple of two sets of names and two sets of sentences.
  - A hypersequent is a set of sequents without sets of names on either side.

CQML

$\varphi \quad \psi \quad \gamma \quad \theta$

Sentences

CQML

φ	ψ	γ	θ
T	F	F	T

CQML

$$\begin{array}{ccc} \varphi, \theta & \Rightarrow & \psi, \gamma \\ T & & F \end{array}$$

CQML

$$\begin{array}{ccc} \Gamma & \Rightarrow & \Sigma \\ T & & F \end{array}$$

CQML

$$\begin{array}{ccccccc} A & : & \Gamma & \Rightarrow & \Sigma & : & B \\ \text{Denotes} & & \text{T} & & \text{F} & & \text{doesn't denote} \end{array}$$

CQML

$$(A : \Gamma \Rightarrow \Sigma : B)$$

$$\mathsf{w}$$

CQML

$$(A : \Gamma \Rightarrow \Sigma : B) \quad ; \quad (C : \Delta \Rightarrow \Lambda : D) \quad ; \quad \dots \quad ; \quad (P : \Pi \Rightarrow \Theta : Q)$$

$$\mathsf{w}_1 \qquad\qquad \mathsf{w}_2 \qquad\qquad\qquad \mathsf{w}_n$$

CQML

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- Equivalent rules for $\mathfrak{Z}$

$$\text{L}\mathfrak{Z}\ \frac{G; (t : \ \Rightarrow \ :); (A : \Gamma, \varphi[t/x] \Rightarrow \Sigma : B); H}{G; (\Gamma, \mathfrak{Z}x\varphi \Rightarrow \Sigma); H}$$

$$\text{R}\mathfrak{Z}\ \frac{G; (C : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \varphi[t/x], \Sigma : B); H \qquad G; (C : \Delta \Rightarrow \Lambda : D, t); (A : \Gamma \Rightarrow \Sigma : B); H}{G; (C : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \mathfrak{Z}x\varphi, \Sigma : B); H}$$

- All the non-quantified rules are the same as before.
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$$\text{LƷ}\ \frac{G; (t : \quad \Rightarrow \quad :); (A : \Gamma, \varphi[t/x] \Rightarrow \Sigma : B); H}{G; (\Gamma, \text{Ʒ}x\varphi \Rightarrow \Sigma); H}$$

$$\text{RƷ}\ \frac{G; (C : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \varphi[t/x], \Sigma : B); H \qquad G; (C : \Delta \Rightarrow \Lambda : D, t); (A : \Gamma \Rightarrow \Sigma : B); H}{G; (C : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \text{Ʒ}x\varphi, \Sigma : B); H}$$

- Rules for $\exists$

$$\text{L}\exists\ \frac{G; (A, t : \Gamma, \varphi[t/x] \Rightarrow \Sigma : B); H}{G; (A : \Gamma, \exists x\varphi \Rightarrow \Sigma : B; H}$$

$$\text{R}\exists\ \frac{G; (A : \Gamma \Rightarrow \varphi[t/x], \Sigma : B); H \qquad G; (A : \Gamma \Rightarrow \Sigma : t, B); H}{G; (A : \Gamma \Rightarrow \exists x\varphi, \Sigma : B); H}$$

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$$\text{R}\exists\ \frac{G; (A : \Gamma \Rightarrow \varphi[t/x], \Sigma : B); H \qquad G; (A : \Gamma \Rightarrow \Sigma : t, B); H}{G; (A : \Gamma \Rightarrow \exists x\varphi, \Sigma : B); H}$$

Fact

CQML proves that $(: \Diamond\exists x\Box\neg Fx \Rightarrow \text{Ʒ}x\neg Fx :)$

ONTOLOGICAL COMMITMENT

Argument

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3. Since $\Diamond\exists x\Box\neg Fx$ entails Ʒ$x\neg Fx$, all the consequences of Ʒ$x\neg Fx$ are consequences of $\Diamond\exists x\Box\neg Fx$.

C. Ʒ$x\neg Fx$ entails no sentences that commits one to there being $\neg F$'s.

Recapturing the Disagreement

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Ex_t

$$\mathsf{Ex}_t \ \frac{G; (C, t : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \Sigma : B); H}{G; (C : \Delta \Rightarrow \Lambda : D); (A, t : \Gamma \Rightarrow \Sigma : B); H}$$

RECAPTURING NECESSITISM

CQML is contingentist: it is sound and complete with respect to $\mathcal{M}_C$.

Ex_t

$$\mathsf{Ex}_t \; \frac{G; (C, t : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \Sigma : B); H}{G; (C : \Delta \Rightarrow \Lambda : D); (A, t : \Gamma \Rightarrow \Sigma : B); H}$$

Call $CQML$+Ex_t $NQML$

Adequacy Fact

CQML is sound and complete with respect to $\mathcal{M}_C$ and *NQML* is sound and complete with respect to $\mathcal{M}_N$

RECAPTURING NECESSITISM

CQML is contingentist: it is sound and complete with respect to $\mathcal{M}_C$.

Ex_t

$$\mathsf{Ex}_t\ \frac{G; (C, t : \Delta \Rightarrow \Lambda : D); (A : \Gamma \Rightarrow \Sigma : B); H}{G; (C : \Delta \Rightarrow \Lambda : D); (A, t : \Gamma \Rightarrow \Sigma : B); H}$$

Call *CQML*+Ex_t *NQML*

Adequacy Fact

CQML is sound and complete with respect to $\mathcal{M}_C$ and *NQML* is sound and complete with respect to $\mathcal{M}_N$

Adequacy Fact holds even before the language has connectives, modal operators, or quantifiers

UPSHOT

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1. BF + CBF is neither necessary nor sufficient to characterize the disagreement between necessitists and contingentists

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2. Arguments about the validity of BF +CBF are only indirectly relevant to the disagreement between necessitism and contingentism

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1. BF + CBF is neither necessary nor sufficient to characterize the disagreement between necessitists and contingentists
2. Arguments about the validity of BF +CBF are only indirectly relevant to the disagreement between necessitism and contingentism
3. The dispute between necessitists and contingentists rests on the behavior of names, not the validity of any sentence.

Thanks!